

Show all necessary work on this paper. Answers without sufficient supporting work will not be graded. Solve all problems algebraically unless otherwise indicated.

Solve the following systems using substitution.

1. $5x - y = 1$
 $-10x + 2y = -2$

$y = 5x + 1$
 $-10x + 2(5x + 1) = -2$
 $-10x + 10x + 2 = -2$
 $0 = 0$

~~no solution~~
 infinite solutions

Solve the following systems using elimination.

3. $\frac{1}{2}x + \frac{1}{3}y = \frac{1}{3}$
 $\frac{3}{2}x + \frac{1}{4}y = -\frac{1}{8}$

$x = -\frac{1}{3}, y = \frac{3}{2}$

$2(\frac{1}{2}x + \frac{1}{3}y = \frac{1}{3})$
 $8(\frac{3}{2}x + \frac{1}{4}y = -\frac{1}{8})$
 $\begin{cases} 3x + 2y = 2 & \text{--- (1)} \\ 12x + 2y = -1 & \text{--- (2)} \end{cases}$
 $-12x - 8y = -8$
 $0 - 6y = -9$
 $y = \frac{3}{2}$
 $x = -\frac{1}{3}$

Solve the following systems of nonlinear equations.

5. $y = x^2 - 7$
 $y = 5x - 13$

$x^2 - 7 = 5x - 13$
 $x^2 - 7 + 13 - 5x = 0$
 $x^2 - 5x + 6 = 0$
 $(x-2)(x-3) = 0$
 $x = 2, x = 3$

$y = (2)^2 - 7$
 $y = 4 - 7$
 $y = -3$
 $y = (3)^2 - 7$
 $y = 9 - 7$
 $y = 2$

$(2, -3)$
 $(3, 2)$

6. $y^2 + x^2 = 25$
 $-(y^2 - 2x^2 = 1)$

$y^2 + 2x^2 = -1$
 $3x^2 = 24$
 $x^2 = 8$
 $x = \pm\sqrt{8}$
 $x = \pm 2\sqrt{2}$

$y^2 + (2\sqrt{2})^2 = 25$
 $y^2 + 8 = 25$
 $y^2 = 17$
 $y = \pm\sqrt{17}$

Sol. Set
 $(2\sqrt{2}, \sqrt{17})$
 $(2\sqrt{2}, -\sqrt{17})$
 $(-2\sqrt{2}, -\sqrt{17})$
 $(-2\sqrt{2}, \sqrt{17})$

$y = \frac{3}{2}x$

2. $\frac{1}{2}x - \frac{1}{3}y = 4$
 $\frac{3}{2}x - y = 0$

$\frac{1}{2}x - \frac{1}{3} \cdot \frac{3}{2}x = 4$
 $\frac{1}{2}x - \frac{1}{2}x = 4$
 $0 = 4$

no solution

parallel lines
 $(-1, -2, 3)$

$(-1, -2, 3)$
 $3x + 2y - z = -10$

4. $x - y + 2z = 7$
 $-x + 3y + z = -2$

please see attached sheet



-1 -2

$$3x + 2y - z = -10$$

$$x - y + 2z = 7$$

$$-x + 3y + z = -2$$

$$-30 - 4y - z = -10$$

$$-7 - z = -10$$

$$-z + 10 = -7$$

$$z = 3$$

$$(1) 3x + 2y - z = -10$$

$$(3) -x + 3y + z = -2$$

$$(3) x - y + 2z = 7$$

$$(2) 2x - 6y - 2z = 4$$

$$x - y + z = 2$$

$$(5) 3x - 7y = 11$$

$$(1) = (4) 2x + 5y = -12$$

$$(3) 3x - 7y = 11$$

$$(4) -\frac{2}{3}x + \frac{5}{1}y = \frac{-12}{1} \cdot \frac{-2}{3}$$

$$-\frac{2}{3}x +$$

(5)

$$-2x + 15y = -36 \cdot -2$$

(4)

(5)

$$\frac{3x}{1} - \frac{7}{1} \left(\frac{-2x - 12}{5} \right) = \frac{11}{1}$$

$$2x + 12$$

$$y = \frac{-2x - 12}{5}$$

$$15x + 14x + 84 = 55$$

$$29x = -29$$

$$-3 + 7y = 11$$

$$y = -1$$

$$-2 + 5y = -12$$

$$5y = \frac{-10}{5}$$

$$y = -2$$

Write the partial fraction decomposition for each of the following.

7. $\frac{-8x-30}{x^2+10x+25}$

$\frac{-8x-30}{(x+5)^2} = \frac{A}{x+5} + \frac{B}{(x+5)^2}$

$-8x-30 = A(x+5) + B$

$x = -5 \rightarrow -40 - 30 = 0 + B$

$B = 10$

$x = 0 \rightarrow -30 = A(5) + 10$

$-40 = A(5)$

$A = -8$

8. $\frac{13x+2}{(3x+1)^2}$

$\frac{13x+2}{(3x+1)^2} = \frac{A}{3x+1} + \frac{B}{(3x+1)^2}$

$13x+2 = A(3x+1) + B$

$x = -\frac{1}{3} \rightarrow$

$\frac{13(-\frac{1}{3}) + 2}{1(-\frac{1}{3}) + 1} = B$

$B = -\frac{7}{3}$

9. $\frac{x^4 - x^3 + 2x - 1}{x(x^2 + 1)^2}$

Please see attached paper.

Perform the given matrix operations for each of the following.

10. $5 \begin{bmatrix} 4 & 9 \\ -2 & 3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} -6 & 12 \\ 4 & -8 \end{bmatrix}$

$\frac{-8x-30}{(x+5)^2} = \frac{-8}{x+5} + \frac{10}{(x+5)^2}$

Please see attached

11. $\begin{bmatrix} 1 & 4 & -7 \\ -2 & 9 & 5 \\ 12 & 0 & -4 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & 3 \\ 5 & 10 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}^{-1}$

13. $\det \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{vmatrix}$

#8

$$3x+1=0$$
$$x = -\frac{1}{3}$$

$$\frac{13x+2}{(3x+1)^2} = \frac{A}{(3x+1)} + \frac{B}{(3x+1)^2}$$

$$13x+2 = A(3x+1) + B$$

$$x = -\frac{1}{3} = \left(\frac{13}{1} \cdot -\frac{1}{3} \right) + \frac{2}{1} = 0 + B$$

$$-\frac{13}{3} + \frac{6}{3} = B$$

$$-\frac{7}{3} = B$$

$$x=0 \rightarrow 13(0)+2 = A(0+1) + -\frac{7}{3}$$

$$2 = A(1) - \frac{7}{3}$$

$$2 + \frac{7}{3} = A$$

$$\frac{6}{3} + \frac{7}{3} = \frac{13}{3} = A$$

$$\frac{13x+2}{(3x+1)^2} = \frac{\frac{13}{3}}{(3x+1)} + \frac{-\frac{7}{3}}{(3x+1)^2}$$

#9

$$\frac{x^4 - x^3 + 2x - 1}{x(x^2 + 1)^2}$$

#9

$$\frac{x^4 - x^3 + 2x - 1}{x(x^2 + 1)^2} = \frac{A}{x} + \frac{Bx + C}{(x^2 + 1)} + \frac{Dx + E}{(x^2 + 1)^2}$$

$$x^4 - x^3 + 2x - 1 = A(x^2 + 1)^2 + (Bx + C)(x(x^2 + 1)) + (Dx + E)(x)$$

$$= A(x^2 + 1)^2 + (Bx + C)(x)(x^2 + 1) + (Dx + E)(x)$$

$$= A(x^4 + 2x^2 + 1) + Bx^4 + Bx^2 + Cx^3 + Cx + Dx^2 + Ex$$

$$= Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 + Cx^3 + Cx + Dx^2 + Ex$$

$$= (A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A$$

$$= (A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A$$

$$x^4 - x^3 + 2x - 1$$

$$A = -1$$

$$B = 2$$

$$A + B = 1$$

$$C = -1$$

$$C = -1$$

$$D = 0$$

$$2A + B + D = 0$$

$$E = 3$$

$$C + E = 2$$

$$A = -1$$

$$\frac{x^4 - x^3 + 2x - 1}{x(x^2 + 1)^2} = \frac{-1}{x} + \frac{2x - 1}{(x^2 + 1)} + \frac{3}{(x^2 + 1)^2}$$

14. If $\det(A) = -6$, what would be the determinant if you switched rows 1 and 3, multiplied the second row by 12, and took the inverse?

15. Use Gaussian Elimination to solve the following system of equations.

$$2x + y + z = -3$$

$$x - 2y + 3z = 6$$

$$x - y - z = 6$$

16. Use the inverse of a matrix to solve the following system of equations.

$$\frac{1}{100}x - \frac{3}{100}y + \frac{1}{20}z = -49$$

$$\frac{3}{100}x - \frac{7}{100}y - \frac{1}{100}z = 13$$

$$\frac{9}{100}x - \frac{9}{100}y - \frac{9}{100}z = 99$$

Use Cramer's Rule to solve the following systems of equations.

$$0.1x + 0.1y - 0.1z = -1.2$$

17. $0.1x - 0.2y + 0.4z = -1.2$

$$0.5x - 0.3y + 0.8z = -5.9$$

$$4x - 3y + 5z = -\frac{5}{2}$$

18. $7x - 9y - 3z = \frac{3}{2}$

$$x - 5y - 5z = \frac{5}{2}$$

Solve the following problems using a system of equations.

19. A factory producing cell phones has the following cost and revenue functions:

$$C(x) = x^2 + 75x + 2688 \text{ and } R(x) = x^2 + 160x$$

What is the range of cell phones they should produce each day so there is a profit? Round to the nearest number that generates profit.

20. A small fair charges \$1.50 for students, \$1 for children, and \$2 for adults. In one day, three times as many children as adults attended the fair. A total of 800 tickets were sold for a total revenue of \$1,050. How many of each type of ticket was sold?