

Show all work. Answers without adequate justification will not receive full credit. Solve problems algebraically whenever possible. Simplify to the lowest terms.

1. (12 pts) Given $P(x) = 2(x-1)(x+1)^2(x+2)$ answer the following (you do not have to multiply this out)

- a) What is the leading term of $P(x)$? $2x^4$
- b) What is the degree of $P(x)$? 4
- c) As $x \rightarrow \infty$ the function $P(x) \rightarrow$ ∞
- d) As $x \rightarrow -\infty$ the function $P(x) \rightarrow$ ∞
- e) How many turning points (bumps) does it have? 3
- f) Write the coordinates of the x-intercept(s) $(-2, 0), (-1, 0), (1, 0)$
 (indicate if they are single/ double / triple etc..zeros)
- g) Find the coordinates of the P-intercept(s) $(0, -4)$
 show work algebraically

$P(x) = 2(0-1)(0+1)^2(0+2) = -4 = y$

$2(-1)(1)^2(2)$
 $-2(1)2$

2. (12 pts) $Q(x) = -3(x+1)^2 + 4$ is a transformed quadratic equation.

- a) Write $Q(x)$ in standard form $Q(x) =$ $-3x^2 - 6x + 1$
 Show work algebraically

$Q(x) = -3(x+1)(x+1) + 4$
 $-3(x^2 + x + x + 1) + 4$
 $-3x^2 - 6x - 3 + 4$

$Q(x) = -3x^2 - 6x + 1$

- b) As $x \rightarrow \infty$ the function $Q(x) \rightarrow$ $-\infty$
- c) As $x \rightarrow -\infty$ the function $Q(x) \rightarrow$ $-\infty$

d) What are the coordinates of the Q intercept(s) $(0, 1)$

e) What are the coordinates of the vertex $(-1, 4)$

f) Is the Vertex a max or a min? Maximum

$x=0$
 $y = Q(x) = -3(0) - 6(0) + 1 = 1$

$h = \frac{-(-6)}{2(-3)} = \frac{6}{-6} = -1 = x$

$Q(x) = -3(-1)^2 - 6(-1) + 1$
 $-3(1) + 6 + 1$
 $-3 + 6 + 1$
 $3 + 1 = 4$ $(-1, 4)$

$h = \frac{-b}{2a}$

3. (12 pts) A resident of planet Zeldok kicks a ball from the top of building, and its height after t seconds is given by $h(t) = -3t^2 + 18t + 9$

a) What was the height of the building?

9 units

b) What is the maximum height the ball reaches?

36 units

$$t = \frac{-18}{2(-3)} = \frac{-18}{-6} = 3 = x_v$$

$$h(3) = -3(3)^2 + 18(3) + 9 = -27 + 54 + 9 = 36 \text{ units}$$

c) When does the ball hit the ground?

$$0 = -3t^2 + 18t + 9$$

$$-3(t^2 - 6 - 3)$$

~~$$-3(t^2 - 6 + 9 - 3)$$~~

~~$$-9 = -3(t-3)^2 + 9$$~~

~~$$= -3(t^2 - 6 - 3)$$~~

~~$$x = \frac{3 \pm \sqrt{9+9}}{-6}$$~~

~~$$x = \frac{-18 \pm \sqrt{18^2 - 4(-3)(9)}}{2(-3)}$$~~

~~$$x = \frac{-18 \pm \sqrt{324 - 108}}{-6}$$~~

~~$$x = \frac{-18 \pm \sqrt{216}}{-6}$$~~

~~$$x = \frac{-18 \pm \sqrt{36 \cdot 6}}{-6}$$~~

4. (6 pts) Given the graph below write the equation for $G(x)$ =

(show work to find the value of the constant, a)

$$G(x) = \frac{1}{3}(x+2)(x-1)(x-3)$$

3c

$$0 = 3x^2 + 18x + 9$$

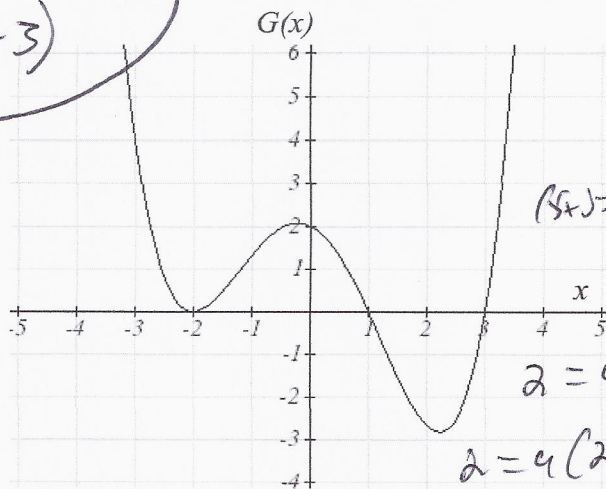
$$= \frac{-18 \pm \sqrt{18^2 - 4(3)(9)}}{2(3)}$$

$$= \frac{-18 \pm \sqrt{432}}{6}$$

-6

$$x = \frac{-18 \pm \sqrt{432}}{-6}$$

-6



$$G(x) = a(x+2)(x-1)(x-3)$$

$$2 = a(0+2)(0-1)(0-3)$$

$$2 = a(2)(-1)(-3)$$

$$2 = a(6)$$

$$2 = 6a$$

$$a = \frac{1}{3}$$

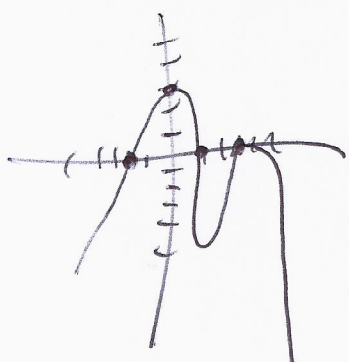
$$G(x) = \frac{1}{3}(x+2)(x-1)(x-3)$$

~~x = 0.464~~ or $x = 6.464$ units of time

5. (10 pts) Find an equation for a polynomial that would pass through the following points, with long run behavior $f(x) = -x^4$.

x	-2	0	1	3
$f(x)$	0	3	0	0

(hint: start by sketching a graph - there is more than one possible answer)



$$f(x) = a(x+2)(x+0)(x-1)(x-3)$$

$$f(x) = -a(x+2)(x-1)(x-3)$$

$$H(x) = \frac{-3(x-2)(x+2)}{(x+3)(x-1)}$$

6. (10 pts) Given the graph below, what is the equation?

(show work to find the value of the constant, a)

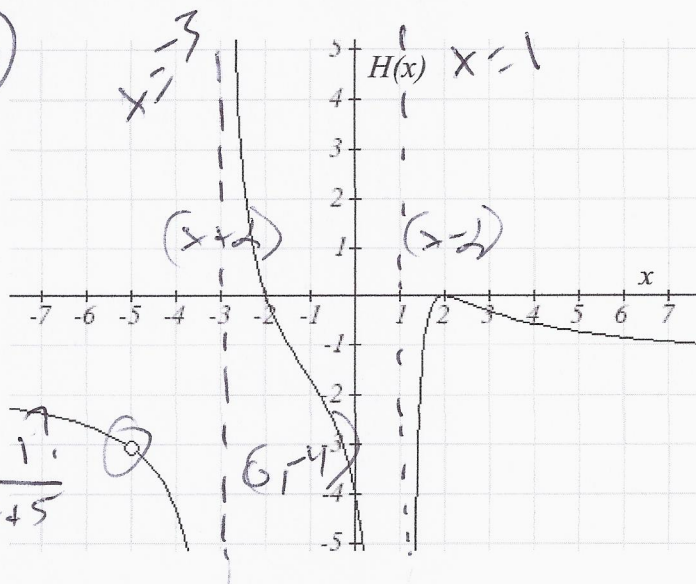
$$f(x) = a(x-2)(x+2)$$

$a = -3$

$$-4 = a(-2)(2) = a(-4) \Rightarrow a = 1$$

$$-4 = a \cdot 4 \Rightarrow a = -1$$

$$a = -3$$



7. (3 pts) Where is the horizontal asymptote (if any) of $m(x) = \frac{3(x+3)(x-2)}{(x+4)^2(x-1)}$

$$= \frac{3(x^2 - 2x + 3x - 6)}{(x+4)^2(x-1)} = \frac{3x^2 + 3x - 18}{x^3 + 8x^2 + 7x - 4}$$

lower degree / higher degree = 0

Hor. Asymp @ $y = 0$

8. (3 pts) Where is the horizontal asymptote (if any) of $n(x) = \frac{x^2 - 6}{5x - 3x^2}$

$$n(x) = \frac{x^2 - 6}{-3x^2 + 5x} = \frac{x^2}{-3x^2} = -\frac{1}{3}$$

same degree

$y = -\frac{1}{3}$

9. (18 pts) Given $j(x) = \frac{2(x-2)(x+1)}{(x+3)^2}$

$= \frac{2(x^2 - 2x + x - 2)}{x^2 + 6x + 9}$
 $= \frac{2x^2 - 2x - 4}{x^2 + 6x + 9}$

a) What are the coordinates of the x-intercept(s)? $(-1, 0), (2, 0)$

b) What are the coordinates of the y-intercept(s)? $(0, -4/9)$

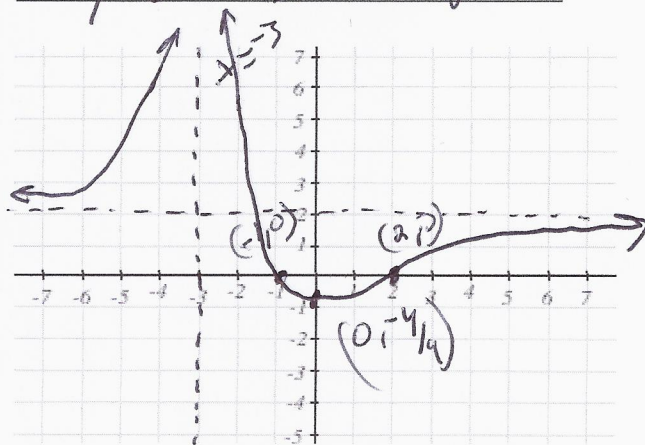
c) Where are the vertical asymptotes (if any)? $x = -3$

d) Where are the horizontal asymptotes (if any)? $y = 2$ same degree

e) As $x \rightarrow \infty, j(x) \rightarrow$ 2

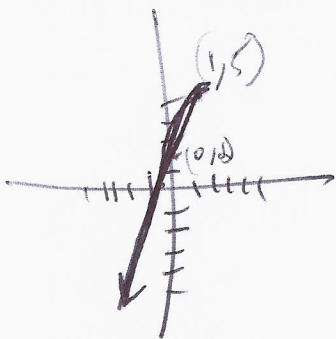
f) As $x \rightarrow -\infty, j(x) \rightarrow$ 2

g) Graph the function & its important features on the grid to the right



hole @ $(-0, 14/9, -0, 45)$?

10. (7 pts) Given $g(x) = -3(x-1)^2 + 5$ on the restricted domain $x \leq 1$, find the inverse of $g(x)$



$g^{-1}(x) =$
 $x = -3(y-1)^2 + 5$
 $x - 5 = -3(y-1)^2$
 $\frac{x-5}{-3} = (y-1)^2$
 $\sqrt{\frac{x-5}{-3}} = y-1$
 $y = 1 + \sqrt{\frac{x-5}{-3}}$

$-\sqrt{\frac{x-5}{-3}} + 1 = y$
 $g^{-1}(x) \{y \leq 1\} =$
 $y = -3(y-1)^2 + 5$

11. (7 pts) Given $k(x) = \frac{4x}{5-3x}$, find $k^{-1}(x)$

$k^{-1}(x) = \frac{5x}{3x+4}$

$3x+4 = 0$
 $3x = -4$
 $x = -4/3$

$g^{-1}(x) = -\sqrt{\frac{x-5}{-3}} + 1, \{x \leq 5\}$

$k^{-1}(-8) = \frac{5(-8)}{3(-8)+4} = \frac{-40}{-24+4} = \frac{-40}{-20} = 2$

$k^{-1}(-8) = \frac{5(-8)}{3(-8)+4} = \frac{-40}{-24+4} = \frac{-40}{-20} = 2$
 $(-8, 2)$ ✓