MATH 171 HARBUCK FINAL EXAM MICHAEL SPRING 2018 7/18/19

1. Complete the multiplication $(2x+3)(x^2-2)$.

$2x^3-2x+3x^2-6$

$\boxed{2x^3+3x^2-2x-6}$

2. Solve the inequality $|x+1|>3$:

$x+1>3$ $\quad x+1<-3$ $\quad (-\infty,-4)\cup(2,\infty)$

$x>2$ $\quad x<-3-1$

$x<-4$ $\quad \{x \mid x>2 \text{ or } x<-4\}$

3. The stopping distance d of an automobile is directly proportional to the square of its speed v. A car required 30 feet to stop when its speed was 40 miles per hour. Find its stopping distance (in feet) when its speed v is 60 miles per hour.

$d=\frac{v^2}{c}$

$30=\frac{40^2}{c}$ $\quad 30c=1600$ $\quad c=\frac{1600}{30}$

$d=\frac{60^2}{53\frac{1}{3}}$ $\quad d@60\text{mph}=67.5\text{ ft.}$

4. If $\frac{1}{2}a-2(a+1)=a$ and $b+\frac{1}{3}(b-1)=1$, find $a+b$:

$\frac{1}{2}a-2a+2+b^2-b+\frac{1}{3}b-b=1$ $\quad b=1-\frac{1}{3}b$

$\frac{1}{2}a-2a-2+2-\frac{1}{3}b$ $\quad \boxed{-\frac{3}{2}a+\left(-\frac{1}{3}b\right)}$ $\quad b=1-\frac{1}{3}(b-1)$

$b=1-\frac{1}{3}b+1$

$b=2-\frac{1}{3}b$

$\boxed{-\frac{3}{2}a-\frac{1}{3}b}$ ✓ simplified

5. Simplify the expression $\left(\frac{a^{-2}}{2bc^{-3}}\right)^{-2}$

$$\frac{a^4}{2^{-2}b^{-2}c^6} = \frac{4a^4b^2}{c^6}$$

6. Find the domain of the function $f(x) = \sqrt{9-x^2}$.

$9-x^2 \geq 0$

$-x^2 \geq -9$

$x^2 \leq 9$

$x \leq \pm 3$

$\{x \mid -3 \leq x \leq 3\}$

$(-3, 3) \cup (3, 0)$

$0 \leq 9 - x^2$

$x^2 \leq 9$

$|x| \leq \sqrt{9}$

$x^2 + y^2 = 9$ Circle w/ limited domain & range

x	-4	-3	0	3	4
y	i	0	3	0	i

7. Which of the following functions is an odd function?

(a) $f(x) = x^2 + 1$ EVEN

(b) $f(x) = 3x^3 - 2x + 5$ ODD, 1 to 1 func.

(c) $f(x) = x^3 - 1$ ODD, 1 to 1

(d) $f(x) = 4x^5 - 2x^3 + x$ ODD

(e) $f(x) = 5x^4 - 3x^2 + 1$ EVEN

8. Perform the indicated operation and simplify: $\frac{2}{x+2} - \frac{x}{x-2}$.

$$\frac{2}{x+2} - \frac{x}{x-2}$$

$$\frac{2(x-2)}{(x+2)(x-2)} - \frac{x(x+2)}{(x+2)(x-2)} = \frac{2x-4-x^2+2}{x^2-4} = \frac{-x^2+4x-4}{x^2-4}$$

9. The graph of the function $y = (x-2)^3 - 3$ can be obtained from the graph of the function $y = x^3$ by which of the following transformations?

   (a) Shift to the right by 3 units, then shift down by 2 units;

   (b) Shift to the left by 4 units, reflect around the y-axis, then shift up by 2 units;

   (c) Shift to the right by 2 units, then shift down by 3 units;

   (d) Shift to the left by 2 units, then shift down by 3 units;

   (e) none of the above.

10. Use the Rational Zero Test to determine which of the following numbers is definitely **not** a zero of the polynomial $3x^4 + 3x^3 + ax^2 + bx - 12$ (where a and b are some integers)?

    (a) -1

    (b) $2/3$

    (c) $-4/3$

    (d) 12

    (e) $3/2$

$-1, -2, 3, -4, 6, -12$, $1, 2, 3, 4, 6, 12$

$-1, -3$ $\quad$ $1, 3$

$\frac{p}{q} = \frac{\pm \text{factor}(p)}{\pm \text{factor}(q)}$

11. A leather jacket was on sale at a department store at 40% off its regular price. The store manager later decided to give it an additional 30% off after the first 40% price cut. The final price of the deeply discounted jacket is now \$147, what was its original price?

$147 = [P - (P, 40)] - 1[(P - (P, 40))(, 30)]$ three hundred and fifty dollars

$= 1P - , 4P - , 3P + P(, 12)$

$147 = 0,42P$

\$350,00

12. Given that $(0, 1)$ is the center of a circle and $(-4, 4)$ is a point on the circle, find the radius of the circle:

$(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-1)^2 = r$

$r = 5$

$(-4-0)^2 + (4-1)^2 = r$

$(-4)^2 + (3)^2 = r^2$

$16 + 9 = 25 = r^2$

13. Solve the equation $e^{2x+b} = 23$ for x:

$\ln e^{2x+b} = \ln 23$

$2x + b = \ln 23$

$x = \dfrac{\ln 23 - b}{2}$

14. Given that $(1, -5)$ and $(3, 7)$ form a pair of diameter points on a circle, find the center of the circle.

$\left(\frac{1+3}{2}, \frac{-5+7}{2}\right)$

Center $= (2, 1)$

$(x-2)^2 + (y-1)^2 = r^2$

$(1-2)^2 + (-5-1)^2 = r^2$

$(-1)^2 + (-6)^2 = r^2$

$1 + 36 = r^2$

$r = \pm\sqrt{37}$

15. Given $f(x) = \sqrt{x^2 + x + 1}$ and $g(x) = x^2 - 1$, find $(g \circ f)(x)$.

$g(f(x)) = x^2 + x + 1 - 1$

$g(f(x)) = x^2 + x$

16. Given $f(x) = \begin{cases} -x^3 + 2x + 1 & \text{if } x < 1 \\ x^2 + 5 & \text{if } x \geq 1 \end{cases}$, find $f(-1) + f(1) + f(2)$.

$x < 1 \rightarrow f(x) = -x^3 + 2x + 1$

$= -(-1)^3 + 2(-1) + 1$

$= 1 - 2 + 1$

$f(-1) = 0$

$x \geq 1 \rightarrow f(x) = x^2 + 5$

$f(1) = (1)^2 + 5$

$f(1) = 6$

$\rightarrow f(2) = (2)^2 + 5$

$f(2) = 4 + 5$

$f(2) = 9$

$f(-1) + f(1) + f(2) = 15$

$f(-1) + f(1) + f(2) = 15$

17. Given $f(x) = 3x - 9$, find $f^{-1}(x)$.

$f(x)=$

$y = 3x - 9$

$f^{-1}(x) = x = 3y - 9$

$f^{-1}(x) = \frac{x-9}{3}$

18. If a and b are positive numbers such that $a^{3/2} = 8$ and $b^{2/3} = 4$, find $b - a$:

$\sqrt{a^3} = 8$ $\quad a^3 = 64$ $\quad \sqrt[3]{b^2} = 4$

$(\sqrt{a^3})^2 = 8^2$ $\quad a = \sqrt[3]{64}$ $\quad b^2 = 4^3$

$a = 4$ $\quad b^2 = 64$

$b = 8$

$b - a = x$

$8 - 4 = 4$

19. The equation of a circle is $x^2 + 6x + y^2 - 2y = 6$, find its radius.

Goes to Standard

$x^2 + 6x + 9 + y^2 - 2y + 1 = 6 + 9 + 1$

$(x+3)^2 + (y-1)^2 = 16$

$(x+3)^2 + (y-1)^2 = 4^2$

$r = 4$

20. Find the vertex of the quadratic function $f(x) = 2x^2 - 16x + 25$.

$y = A(x-h)^2 + k$

$h = -\frac{b}{2a}$

$h = \frac{16}{2(2)}$ $\quad h = 4$

$k = \frac{4ac - b^2}{4a}$

$k = 4(2)25 + 16^2/4(2)$

$k = 200 - 256$

8

$k = -56 - 7$

$2(4)^2 - 16(4) + 25$

$k = -7$

$y = 2(x-4)^2 - 7$

VERTEX = $(4, -7)$

21. Determine which interval given below is the solution set of the inequality $(2x-1)^2 \leq 9$.

(a) $[-1, 1]$

(b) $(-1, 2]$

(d) $(-\infty, -1) \cup (2, \infty)$

(e) $[-1, 2]$

$4x^2 - 2x - 2x + 1 \leq 9$

$4x^2 - 4x - 8 \leq 0$

$4(x^2 - x - 2) \leq 0$

$4(x-2)(x+1) \leq 0$

$x = 2$

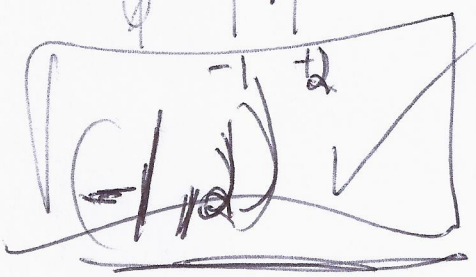
$x = -1$

	-2	0	3
	≤ 0	$-8 \leq 0$	$16 \not\leq 0$
	no	✓	no

$4(-2-2)(-2+1) \leq 0$?

$4(0-2)(0+1) \leq 0$?

$4(3-2)(3+1) \leq 0$



General Form of rational inequalities-

22. Solve the inequality $\frac{3}{x-1} \leq -2$ and express your answer in interval notation:

$3 \leq -2(x-1)$

$3 \leq -2x+2$

$3-2 \leq -2x$

$1 \leq -2x$

$x \leq -\frac{1}{2}$

$x \neq 1$

$\frac{3}{x-1} + 2 \leq 0$

$\frac{3}{x-1} + \frac{2(x-1)}{x-1} \leq 0$

$\frac{3+2x-2}{x-1} \leq 0$

$\frac{2x+1}{x-1} \leq 0$

$2x+1 = 0 \quad x-1 = 0$

$x = -\frac{1}{2} \quad x = 1$

$\{x \mid x \leq -\frac{1}{2}, x < 1\}$

$\left(-\frac{1}{2}, 1\right)$

23. Which of the following **BEST** describes the solution(s) of the equation $3x^2 - 11x + 7 = 0$?

$x = \frac{11 \pm \sqrt{121 - 4(3)7}}{2(3)} \qquad 2(3)$

$x = \frac{11}{6} \pm \frac{\sqrt{37}}{6}$

(a) There are two solutions and both are positive.

(b) There are two solutions, one is positive and the other is negative.

(c) There is only one solution and it is positive.

(d) There is only one solution and it is negative.

(e) There are no real solutions to the equation.

24. Find the **horizontal asymptote**, if any, of the function $f(x) = \frac{x^2 - 3x + 2}{2x^2 - 32}$.

polys same degree so

Horizontal Asymptote $= y = \frac{1}{2}$

25. The vertex of a quadratic function is $(2, -1)$ and its y-intercept is 7. Find the function.

$y = A(x-h)^2 + k$

$y = A(x-2)^2 - 1$

$7 = A(0-2)^2 - 1$

$7 = A(2)^2 - 1$

$7 = A(4) - 1$

$8 = 4A$

$A = 2$

$y = 2(x-2)^2 - 1$

$f(x) = 2(x-2)^2 - 1$

26. Which of the following is a factor of the polynomial $6x^2 + 11x - 10$?

(a) $2x - 5$
(b) $3x - 5$
(c) $6x - 1$
(d) $3x - 2$
(e) $6x - 5$

27. Combine $2\log(4x) + 3\log y - 4\log z$ into a single logarithm term:

28. Solve the equation $3x^2 - 7x + 3 = 0$ using the quadratic formula:

29. Find the slope of the line that goes through the points $(-3, 5)$ and $(-5, 9)$.

(a) 2
(b) 1
(c) 0
(d) -1
(e) -2

30. Find the **range** of the function $f(x) = -2e^{x-3} + 4$.

31. Solve the equation $3^{2x+1} = 9^{-x+2}$

32. The polynomial $f(x) = 3x^{99} - 2x^{25} + 5x + 1$ is divided by $x + 1$. Use the remainder theorem to find the remainder.

33. If $f(x)$ is an even function and $f(-2) = -3$, then which of the following must be true?

(a) $f(-3) = 2$

(b) $f(2) = -3$

(c) $f(2) = 3$

(d) $f(3) = -2$

(e) $f(-3) = -2$

34. Find the quotient of $\dfrac{3x^3 + x^2 - 5x + 1}{x - 1}$ by either the synthetic division or the long division.

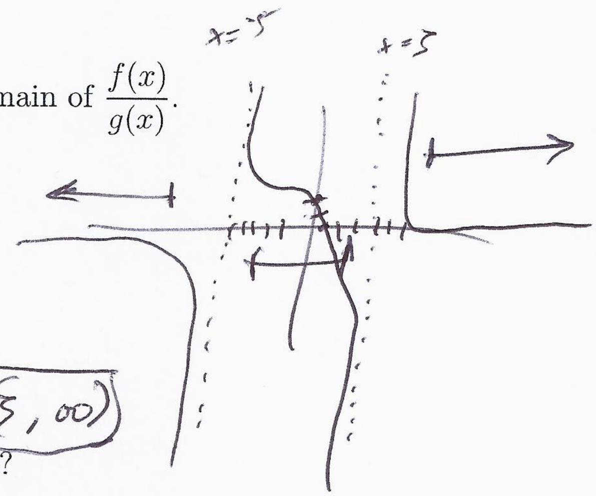
1 | 3 1 -5 1
3 4 -1
3 4 -1 0

$\dfrac{3x^3 + x^2 - 5x + 1}{x-1} = 3x^2 + 4x - 1$

35. Given $f(x) = x - 2$ and $g(x) = x^2 + 2x - 15$, find the domain of $\dfrac{f(x)}{g(x)}$.

$\dfrac{x-2}{x^2+2x-15} = \dfrac{(x-2)}{(x+5)(x-3)}$ (0, 2) (0.13, 0)

$x = 0 \to \dfrac{-2}{5(-3)} = \dfrac{-2}{-15} = \dfrac{2}{15} \approx 0.1333$



Domain $= (-\infty, -5) \cup (-5, 3) \cup (3, \infty)$

36. $\log_5(13)$ is the solution to which of the following equations?

(a) $5x = 13$
(b) $13^x = 5$
(c) $x^5 = 13$
(d) $5^x = 13$
(e) $13^x = 5$

37. Solve the equation $\log_5(2x + 1) = 2$.

$\log_5 2x+1 = 2$

$5^2 = 2x+1$

$-1 + 25 = 2x + 1 - 1$

$24 = 2x$

$x = 12$ ✓

$f(12) = \log_5 2(12) + 1 = 2$

$= 5^2 = 2(12) + 1$

$25 = 25$ ✓

38. Carl opened a savings account with an initial deposit of \$10,000. The account has a fixed APR of 4.5% and the interest is compounded monthly. If Carl makes no further deposit, what will be the account's accumulated value after 5 years (round off to the nearest cent)?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 10000\left(1+\frac{0.045}{12}\right)^{12(5)}$

$= 10000(1.00375)^{60}$

$= 10000(1.25179)$

$A =$ \$12,517.96

39. If x and y are the solutions to the system of equations $\begin{cases} x - y = 3 \\ 3x + y = 5 \end{cases}$, find the value of $x^2 + y^2$:

$-y = -3 + x$

$y = x - 3$

$y = (2) - 3$

$y = -1$

$3x + x - 3 = 5$

$4x - 3 = 5$

$4x = 8$

$x = 2$

$(2, -1)$

$x^2 + y^2 =$

$2^2 + (-1)^2 =$

$4 + 1 = 5$

40. A line passes through the point $(-3, 4)$ and is perpendicular to the line whose equation is $y = -\frac{3}{2}x + 5$, its equation is:

$y = \frac{2}{3}x + 5$

$4 = \frac{2}{3}(-3) + b$

$\frac{4}{1} = \frac{-6}{3} + b$

$\frac{4}{1} + \frac{2}{1} = b$

$f(x) = \frac{2}{3} + 6$

$y = \frac{2}{3} + 6$

End of Exam